Notes

Use of Cavitation Tendency Ratios for Predicting Suction Specific Speed

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A simple derivation is presented to extend the use of the well-known pump similarity laws to include the concept of the cavitation tendency ratio. A more accurate prediction of centrifugal pump net positive suction head requirements should result. Extrapolation of allowable suction specific speeds for various fluids or a single fluid at various conditions of state can reduce pump testing hazards and costs, particularly for cryogenic, corrosive, or toxic fluids.

Nomenclature

B'	= cavitation tendency number, Btu/lb
CTR	= cavitation tendency ratio
C_p	= specific heat of liquid at constant pressure, Btu/lb-°F
H	= pump head, ft
H_{sv}	= pump suppression head, ft
N, n	= speed, rpm
N_s	= specific speed = $N(Q)^{1/2}/(H)^{3/4}$
$NPSH$	= net positive suction head, ft
Q	= pump flow, gal/min or ft³/min
S	= suction specific speed = $N(Q)^{1/2}/(H_{sv})^{3/4}$
$\partial T/\partial P$	= rate of change of temperature with respect to vapor pressure, °F/lb/in.²
V_l	= specific volume of liquid, ft³/lb
V_g	= specific volume of gas, ft³/lb
λ	= latent heat of vaporization, Btu/lb
σ	= Thoma-Moody cavitation parameter = H_{sv}/H

THE use of similarity considerations for the determination of pump performance has been employed now for a considerable number of years. Its use in predicting "prototype" pump performance from model tests is well established.[1–5] However, it has long been known that, with respect to accurate close tolerance predictions of net positive suction head requirements for high performance chemical pumps of the type used for missile rocket engine turbo-pumps, these equations considerably over-simplify the cavitation inception phenomenon. Wislicenus,[6] Stepanoff and Stahl,[7] and others[8–12] have discussed this area and proposed various concepts for a better understanding of the cavitation mechanism. Recently, R. B. Jacobs presented a theory[13] that attempts further to extend our knowledge of this complex area. The author has attempted to relate a portion of this concept to the existing similarity laws.

Presented herein is essentially a condensation of a broader concept concerning the use of turbo-boost pumps in missile booster and space vehicle propulsion systems.[14] Similarity in centrifugal pump performance is given by the well-known expression for specific speed, namely,

$$N_s = n(Q)^{1/2}/(H)^{3/4} = \text{const} \tag{1}$$

At constant specific speed, prediction of cavitation inception from previous test data can be made by applying the Thoma-Moody parameter, which is given by

$$\sigma = H_{sv}/H = \text{const} \tag{2}$$

An additional relationship, applicable within the restrictions of the similarity laws (maintenance of geometric, kinematic, and dynamic similarity from model to prototype pump), is that of suction specific speed,

$$S = n(Q)^{1/2}/(H_{sv})^{3/4} \tag{3}$$

Since the two parameters σ and S have parallel use in the same field of application, it is desirable to state an analytic expression for the relation between them. This expression, which has been derived previously from the explicit expressions for S and N_s, is

$$\sigma = (N_s/S)^{4/3} \tag{4}$$

The preceding analyses for cavitation inception are incomplete in that the effects of thermodynamic, heat transfer, and fluid properties (other than density and viscosity) are not taken into consideration. Various theoreticians and experimentalists (see Refs. 6–8 and 10) have endeavored to provide analytical or empirical evaluations so that extrapolation from existing cavitation similarity laws would be possible. That they have been only moderately successful is an indication of the highly complex aspects of the problem. Additional variables that definitely appear to affect cavitation inception are surface tension, latent heat of vaporization, vapor/liquid volume ratio, thermal conductivity, heat capacity, bubble dynamics, compressibility, rate of change of vapor pressure with temperature, internal pump hydraulics, time, and contamination. Such a formidable array of variables has made formulation of an extension of the Thoma-Moody cavitation parameter and suction specific speed concept extremely difficult. However, recent theoretical and experimental work[13] conducted by R. B. Jacobs at the National Bureau of Standards has led to the concept of a "cavitation tendency ratio" (CTR). This is not to be confused with the "cavitation tendency number" B of Ref. 7, defined as the "vapor-to-liquid-volume" ratio created by flashing due to a small pressure drop below initial saturation pressure. The CTR concept is based on determining the suction head depression required to create a sufficient volume of vapor to produce incipient cavitation conditions at the inlet to a pump, but not appreciably affecting pump capacity and head generation.

The cavitation tendency ratio is defined as follows:

$$CTR = B_A'/B_B' = NPSH_B/NPSH_A \tag{5}$$

and where

$$B' = \frac{\lambda \times (V_l/V_g)}{1 - [(C_p/V_l)(\partial T/\partial P)(778/144)]} \text{ Btu/lb} \tag{6}$$

As can be seen by reviewing Eq. (6), only a portion of the additional variables listed previously has been taken into consideration. Thus precise correlation of this relationship with experimental results would not be anticipated. However, surprisingly close results were obtained in tests at the National Bureau of Standards and to a less precise degree in Ref. 8. These are shown in Table 1. For purposes of this paper, the author has included the theoretical CTR for water and liquid hydrogen, not included in the original National Bureau of Standards tabulation of Ref. 13. Calculations for these values are given in the Appendix of Ref. 14.

Assuming that one accepts the concept of the cavitation tendency ratio as defined previously, it is now a simple matter to relate this ratio to the previously defined suction specific speed. It can be shown using Eq. (4) slightly rearranged that

$$S^{4/3} = (N_s)^{4/3}/\sigma = (N_s)^{4/3}\, H/H_{sv} = \text{const} \tag{7}$$

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Table 1 Comparison of theoretical and experimental cavitation tendency ratios

	Theoretical cavitation tendency ratio (CTR)	Experimental cavitation tendency ratio (CTR)
$\frac{NPSH \text{ (Freon 11 at 85°F)}}{NPSH \text{ (Freon 11 at 120°F)}}$	2.96	1.21[a]
$\frac{NPSH \text{ (}H_2O\text{ at 250°F)}}{NPSH \text{ (}H_2O\text{ at 300°F)}}$	4.1	1.28[a]
$\frac{NPSH \text{ (}H_2O\text{ at 250°F)}}{NPSH \text{ (Freon 11 at 120°F)}}$	1.80	1.31[a]
$\frac{NPSH \text{ (}LN_2\text{)}}{NPSH \text{ (}LH_2\text{)}}$	55.0	42.0[b]
$\frac{NPSH \text{ (}LO_2\text{)}}{NPSH \text{ (}LN_2\text{)}}$	1.37	1.51 to 2.24[b]
$\frac{NPSH \text{ (}LO_2\text{)}}{NPSH \text{ (}LH_2\text{)}}$	75.5 (Jacobs) to 123.5 (Boretz); see Appendix, Ref. 14	64.0 to 94.0[b]
$\frac{NPSH \text{ (}H_2O\text{ at 60°F)}}{NPSH \text{ (}LH_2\text{ at }-423.3\text{°F)}}$	3320; see Appendix, Ref. 14	data not available
$\frac{NPSH \text{ (}H_2O\text{ at 60°F)}}{NPSH \text{ (}LO_2\text{ at }-297.0\text{°F)}}$	26.7; see Appendix, Ref. 14	data not available

[a] 3% drop in head rise across pump (based on Ref. 8).
[b] Conducted by R. Jacobs at National Bureau of Standards, Boulder, Colo., and based on cavition inception principle.

which for a constant specific speed and pump developed head reduces to

$$S_1^{4/3}H_{sv_1} = S_2^{4/3}H_{sv_2} \qquad \text{or} \qquad H_{sv_1}/H_{sv_2} = (S_2/S_1)^{4/3} \tag{8}$$

but

$$H_{sv_1}/H_{sv_2} = NPSH_B/NPSH_A = CTR \tag{9}$$

therefore,

$$CTR = (S_2/S_1)^{4/3} \tag{10}$$

where subscripts 1 and 2 or A and B refer to different fluids or the same fluid at different "state" conditions.

By determining the theoretical cavitation tendency ratio between two fluids and using Eq. (10), it is possible to make more accurate predictions about the allowable suction specific speed of one fluid when test data are available with respect to cavitation inception of the other fluid. As an example, it can be seen from Table 1 that the CTR between water at 60°F and liquid hydrogen is 3320. Thus if a suction specific speed of 10,000 conservatively is assumed as achievable with water, the extrapolated value for the suction specific speed for liquid hydrogen is

$$\begin{aligned} S_{2(LH_2)} &= S_{1(H_2O)} \times (CTR)^{3/4} \\ &= 10{,}000 \times (3320)^{3/4} \\ &= 4{,}300{,}000 \end{aligned}$$

Conclusions

The phenomenon of cavitation is dependent on a multitude of variables. Criteria for defining cavitation inception, operation in the incipient cavitation region, and the assessment of the degree to which one can operate in the cavitation region without experiencing any impeller erosion are not established fully or wholly understood. However, the use of pumps in the missile and space fields will put increased emphasis on obtaining an early solution to these areas of incomplete knowledge. In spite of the foregoing, the following conclusions can be drawn:

1) The use of the simple relationship given in Eq. (10) should prove useful in more accurately estimating allowable suction specific speeds for centrifugal pumps when extrapolating from one fluid to another.

2) In view of the discrepancies existing between theoretical and experimental cavitation tendency ratios (as shown in Table 1), further investigations are required. However, conservative estimates can be made and more rational missile system requirements established.

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